CONCENTRATION OF MEASURE AND ITS APPLICATIONS

by Maxim Raginsk and Igal Sason

During the last two decades, concentration of measure has been the subject of exciting developments in various areas, including convex geometry, functional analysis, statistical physics, high-dimensional statistics, pure and applied probability theory, information theory, theoretical computer science, and learning theory.

One common theme that emerges in these fields is probabilistic stability: complicated, nonlinear functions of a large number of generally dependent random variables often tend to concentrate sharply around their expected values. And yet, even though information theory is only one field in the long list of disciplines that benefit from the concentration phenomenon, it also emerges as the main tool for deriving concentration inequalities — indeed, both the entropy method and the approach based on transportation-information inequalities are two major information-theoretic paths toward proving concentration.

The first explicit appeal to the concentration phenomenon in information theory dates back to the 1970s work by Ahlswede and collaborators, who used the blowing-up lemma for deriving strong converses for a variety of communications and coding problems. In mid-1980's and on, through the pioneering work of Katalin Marton on information-transportation inequalities (which has been recognized with the 2013 Claude E. Shannon Award), mathematicians working on the concentration phenomenon have recognized that information-theoretic techniques often give the sharpest concentration inequalities. More recently, with the attention of the IT community captured by finite-blocklength performance of source and channel codes, concentration inequalities are emerging as one of the key tools for nonasymptotic analysis of source and channel codes.

This tutorial will introduce information theorists to both basic and advanced techniques for deriving concentration inequalities: the martingale method, the entropy method, and the transportation-cost inequalities. Examples from signal processing, communications, source coding, and channel coding will be used throughout the tutorial to illustrate the main ideas.

About the speakers. Maxim Raginsky and Igal Sason have been working for several years on the topic of concentration-of-measure inequalities, as well as on their applications in information theory. They have co-authored a monograph on concentration inequalities that was specifically aimed at information theorists. This monograph was published as part of the Foundations and Trends in Communications and Information Theory in 2013 (volume 10, no. 1–2), and a second edition has been published in 2014. This monograph exemplifies the use of information-theoretic tools for proving powerful concentration inequalities, as well as the applications of concentration inequalities in information theory, communications and coding. It also includes various original proofs and results that have not been published earlier.